*Deriving the likelihood for GzLMM’s*

* Let *h*(**b***i*) and *f*(**y***i*) denote the pdf’s of the random effects and responses for subject *i*, respectively. Also, let *l*(**y***i* | **b***i*) denote the conditional pdf of the responses given the random effects that is a member of the exponential family (e.g., Poisson, binomial, geometric, gamma). Then, we can express the density of the responses as

 for subjects *i*=1,…,*n*.

This will be useful in setting up a likelihood equation for estimation of parameters.

* When subjects are assumed to be independent (the standard case), then the likelihood function is .
* Let’s consider the likelihood function for the GzLMM that has a binary outcome and a random intercept for subjects. Specifically, let’s consider the following model

 for *i*=1,…,*n* and *j*=1,…,*r*





* Note that 
* (For the following, conditioning on covariates and parameters is suppressed for convenience.) This is essentially taken from McCulloch’s *An Introduction to Generalized Linear Mixed Models*. I am starting with the ‘data wide’ model, where subjects are stacked into the Y matrix:
* 







* Note that for =1, we have  and for =0 we have , so collectively we can write .
* Hence we can write









* We could also express the likelihood in a slightly more succinct form in terms of the  (e.g., see Parzen 2011):



* Note that in order to maximize the likelihood we input the data (*x* and *y* values). For example, say that one subject has 5 observed *y* values of (0, 1, 1, 0, 1), with associated *x* values of (1, 2, 3, 4, 5). Then on the interior of the likelihood that is within the square brackets on the last equation (for that subject ‘*i*’), we would have:







We integrate over this with respect to *bi*, then repeat for all subjects and multiply together. This gives a flavor of the complexity of the integration…